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Ergodic Capacity of Generalized Fading Channels With Mobility

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ABSTRACT In this paper, we consider the ergodic channel capacity (ECC) of several generalized fading channels for a mobile user that moves inside a coverage area according to the random waypoint (RWP) mobility model. The ECC with RWP mobility provides the average capacity of a mobile receiver taking into account the probability density function (PDF) of the distance between the user and the transmitting antenna, the path loss exponent and the fading channel. It turns out that the ECC in a mobile environment can be written in terms of the corresponding ECC in the static case. Therefore, we use analytical expressions of the ECC of different generalized fading models for the static channel to derive the corresponding analytical expressions for the ECC in the mobile case. Asymptotic expressions, in the high signal-to-noise ratio (SNR) regime, are also provided that quickly converge to the exact ones. Monte Carlo simulation is also used to show the validity of the derived expressions. The analytical exact and asymptotic expressions enable the system designer to quantify the effect of different physical channel and mobility parameters on the attained average capacity of generalized fading channels.

INDEX TERMS $\alpha - \mu$ fading, $\alpha - \eta - \mu$ fading, $\alpha - \kappa - \mu$ fading, $\alpha - \kappa - \mu$ shadowed fading, ergodic channel capacity, generalized fading models, random waypoint (RWP) mobility model.

I. INTRODUCTION

Channel capacity is a core performance metric in conventional and emerging wireless communication systems. The most widely used capacity measure is the ergodic channel capacity (ECC), which is the maximum transmission rate that is attained over a bandlimited channel assuming channel state information (CSI) knowledge only at the receive side and a fixed transmit power. When fading and shadowing are present, in order to evaluate the channel capacity an average over the fading/shadowing distribution is required. It follows that the ECC is largely affected by the fading conditions incurred during wireless transmission.

Most of the well known short-term fading distributions have been derived assuming a homogeneous scattering environment, resulting from randomly distributed point scatterers. In order to model a non-homogeneous diffuse scattering field, which results from the physical observation that reflecting surfaces are usually spatially correlated, the fading distributions of $\eta - \mu$ and $\kappa - \mu$ have been presented in [1]. The $\kappa - \mu$ distribution models small-scale variations of the fading channel with a line-of-sight (LOS) component, whereas the $\eta - \mu$ assumes no LOS component. Parameter κ accounts for the ratio of the total power of the dominant components to the total power of the scattered waves, η is the power ratio between the inphase and quadrature scattered wave components in Format 1 or it denotes the correlation between the powers of the inphase and quadrature scattered waves in Format 2, and μ denotes the number of multipath clusters which compose the received signal.

The non-linearity of the propagation medium was addressed by the $\alpha - \mu$ fading distribution [2], where parameter α is used to account for the non-linearity of the propagation medium. As a consequence, the fading distributions of $\alpha - \eta - \mu$ and $\alpha - \kappa - \mu$ were introduced in [3] that include Rayleigh, Nakagami-*m*, Weibull, Rice, Hoyt, $\alpha - \mu$, $\eta - \mu$, $\kappa - \mu$ as special cases.

Another extension to signal variations at the wireless receiver are the composite fading models that include the effect of shadowing in the received signal. The generalized-K distribution is an analytically tractable composite fading model that corresponds to a Nakagami/Gamma distribution and is controlled by shaping parameter m for the multipath fading and k for the slow fading of the received power due to shadowing [4]. Similarly, the multipath fading component may follow the κ - μ distribution, while the shadowing of the dominant component follows a Nakagami-m distribution in the κ - μ shadowed composite fading model [5]. An alternative approach is to use the mixture gamma (MG) distribution to closely approximate the composite $\kappa - \mu$ /Nakagami-*m* fading model [6]. Moreover, the $\alpha - \kappa - \mu$ shadowed distribution [7] has been proposed as a generalized fading/shadowing model that results when all the dominant components fluctuate according to a normalized Nakagami-m distribution.

In the literature, the ECC has been derived for the aforementioned generalized fading models in the static case, i.e., assuming a fixed distance between the transmit antenna and the receiver, which yields a fixed average received SNR for the fading distribution. However, mobility is present in most wireless communication networks. The random waypoint (RWP) mobility model has been frequently used in wireless system simulations [8], in scenarios where the nodes move independently to a randomly chosen destination with a randomly selected velocity. According to the theory of random processes, the RWP process has mean-ergodic property. Therefore, the distribution of the distance between transmitter and receiver under the RWP mobility can be obtained. It follows that, for all network dimensions, the exact probability density function (PDF) of the distance has a polynomial form [9].

The impact of RWP mobility on the outage and average bit error rate (BER) performance of wireless receivers appeared for the first time in [9] for the Rayleigh fading channel and was extended to Nakagami-*m* fading in [10]. Recently, more works have appeared in this topic in order to investigate the underlying connections between different physical channel parameters, path loss exponent and mobility. The work in [11] derived the PDF and cumulative distribution function (CDF) of the SNR under RWP mobility for a set of LOS fading distributions, such as κ - μ and κ - μ shadowed fading models. Based on these results, [12] derived analytical expressions for the performance of non-homogeneous channels such as η - μ , κ - μ and κ - μ shadowed fading models in terms of outage probability, BER and ECC. Finally, the work in [13] derived analytical results for the outage and BER performances for the η - μ fading model.

The ECC with mobility is an important key performance indicator (KPI) because it gives the average capacity of the mobile receiver inside a coverage area of radius D taking into account the PDF of the distance between the user and the transmitting antenna, the path loss exponent and the fading distribution. The results can also be used by the system designer as an approximation of the average spectral efficiency (in bits/sec/Hz) and, therefore, the average capacity, when multiplied by the available bandwidth, offered by the base ¹⁶ station located at the center of the coverage area. Moreover, different from previous works, we consider fading models that include both channel non-linearity and non-homogeneity such as $\alpha - \mu$, $\alpha - \eta - \mu$, and $\alpha - \kappa - \mu$ fading models, as well as composite fading such as the generalized-*K* and the $\alpha - \kappa - \mu$ shadowed fading models. The main contributions of this paper are summarized below:

- Based on available analytical expressions for the ECC of a static user over different fading models, such as generalized-*K*, η μ, α μ, α η μ, and α κ μ, we derive analytical results for the ECC with RWP mobility. The expressions are obtained in terms of the well-known Meijer-G function [14], which is available in most popular computing software.
- We derive new exact expressions for the ECC over the α - κ - μ shadowed fading channel, in both static and mobile scenarios.
- We derive a general expression for the asymptotic ECC with RWP mobility as the summation of the asymptotic ECC in the static case and a term that depends only on mobility parameters and the path loss exponent.
- We derive new analytical results for the capacity loss due to fading [15] for all the aforementioned fading distributions that enable us to evaluate the asymptotic ECC in the high SNR regime for both the static and mobile scenarios.

The rest of the paper is organized as follows. In Section II, we derive analytical expressions for the ECC of generalized fading channels under the RWP mobility model, while simplified asymptotic analysis is presented in Section III for the high SNR regime. Numerical and simulation results are presented and discussed in Section IV, while concluding remarks are given in Section V.

II. ERGODIC CAPACITY OF FADING CHANNELS WITH MOBILITY

The ECC of a fading channel with additive white Gaussian noise (AWGN) for a static user whose received signal's instantaneous SNR, X, has PDF $f_X(\cdot)$ and average value \bar{X} , is given by

$$C_s(\bar{X}) = W \int_0^\infty \log_2(1+x) f_X(x) dx, \qquad (1)$$

where *W* is the transmitted bandwidth. Analytical expressions for (1) have been derived in the literature for different fading distributions. The average SNR at distance *R* from the transmitting antenna is given by $\bar{X} = P_t R^{-a}/N$, where P_t is the transmit power, *a* is the path loss exponent, and *N* is the received noise power. In case of user mobility, the average SNR, \bar{X} is a random variable since the distance *R* between the transmitting antenna, located at the center of the coverage area, and the mobile receiver varies. The PDF of the distance *R* for the RWP mobility model is given by [9]

$$f_R(r) = \sum_{k=1}^n \frac{B_k}{D^{\beta_k + 1}} r^{\beta_k}, \qquad 0 \le r \le D, \qquad (2)$$

where n, B_k , and β_k (for k = 1, ..., n) are parameters that depend on the network dimension and D is the radius of the coverage area. It is shown in [11, eq. (15)] that, given the normalized static PDF $f_X(\cdot)$, the PDF of the SNR of the mobile receiver under the RWP model, γ , is given by

$$f_{\gamma}(\gamma) = \sum_{k=1}^{n} \frac{B_k}{a} \bar{\gamma}_D^{\frac{\beta_k+1}{a}} \gamma^{-\left(1+\frac{\beta_k+1}{a}\right)} I_X\left(\frac{\beta_k+1}{a}, \frac{\gamma}{\bar{\gamma}_D}\right), \quad (3)$$

where $\bar{\gamma}_D = P_t D^{-a}/N$ is the average SNR of the mobile receiver at the edge of the coverage area. In (3), the PDF of γ is given in terms of the incomplete moment, $I_X(v, x) = \int_0^x u^v f_X(u) du$, of the static PDF $f_X(x)$ with average SNR $\bar{X} = 1$. Moreover, by employing $t = u\bar{\gamma}_D/\gamma$, (3) can be re-expressed using the PDF of X, as

$$f_{\gamma}(\gamma) = \frac{1}{a\bar{\gamma}_D} \sum_{k=1}^n B_k \int_0^1 t^{\nu_k} f_X\left(\frac{\gamma}{\bar{\gamma}_D}t\right) dt, \qquad (4)$$

where $v_k = \frac{\beta_k + 1}{a}$, k = 1, ..., n. The corresponding CDF of γ may be obtained by integrating (4) to give

$$F_{\gamma}(\gamma) = \frac{1}{a} \sum_{k=1}^{n} B_k \int_0^1 t^{\nu_k - 1} F_X\left(\frac{\gamma}{\bar{\gamma}_D}t\right) dt.$$
(5)

We note from (4) and (5) that the distribution of the SNR for a mobile user can be obtained as an incomplete moment of the corresponding static normalized SNR distribution. Based on this fact, the average capacity for a mobile receiver can be obtained in terms of the average capacity in the static case by inserting (4) in (1) and yields to

$$C_m(\bar{\gamma}_D) = \frac{W}{\ln(2)a} \sum_{k=1}^n B_k \int_0^1 t^{\nu_k - 1} C_s\left(\frac{\bar{\gamma}_D}{t}\right) dt.$$
(6)

In the following sections we present analytical results for the exact ECC of generalized fading distributions in the static and mobile cases.

A. GENERALIZED-K FADING CHANNEL

The generalized-K distribution characterizes the combined effect of multipath and shadow fading on the received signal. The PDF of the instantaneous SNR for a static receiver is given by [4]

$$f_X(x) = \frac{2x^{(\beta-1)/2}}{\Gamma(m)\Gamma(k)} \left(\frac{mk}{\bar{X}}\right)^{(\beta+1)/2} K_\lambda\left(2\sqrt{\frac{mkx}{\bar{X}}}\right), \quad (7)$$

where *m*, *k* are the shaping parameters related to the severity of the small and large scale fading, respectively, $\lambda = k - m$, $\beta = k + m - 1$, and $K_n(\cdot)$ is the *n*-th order modified Bessel function of the second kind [14]. The average channel capacity in the static case is given by [4]

$$C_{s}(\bar{X}) = \frac{W}{\ln(2)\Gamma(m)\Gamma(k)} G_{2,4}^{4,1}\left(\frac{mk}{\bar{X}}\Big|_{\frac{\beta+\lambda+1}{2},\frac{\beta-\lambda+1}{2},0,0}^{0,1}\right), \quad (8)$$

where $G_{p,q}^{m,n}(\cdot)$ is the Meijer's G-function [14, eq. (9.301)]. For the mobile case, substituting (8) in (6) and solving the resulting integral with the help of [14, eq. (7.811.2)], the average capacity is given by

$$C_{m}(\bar{\gamma}_{D}) = \frac{W}{\ln(2)\Gamma(m)\Gamma(k)a} \times \sum_{k=1}^{n} B_{k}G_{3,5}^{4,2}\left(\frac{mk}{2\bar{\gamma}_{D}}\Big|_{\frac{\beta+\lambda+1}{2},\frac{\beta-\lambda+1}{2},0,0,-v_{k}}^{1-v_{k},0,1}\right).$$
 (9)

B. $\eta - \mu$ FADING CHANNEL

The η - μ distribution models a fading channel with no LOS component, with η being the power ratio (in Format 1) or the correlation (in Format 2) between the in-phase and quadrature scattered wave components and μ denotes the number of multipath clusters. The PDF of the instantaneous SNR in the static case is given by [1], [16]

$$f_X(x) = \frac{2\sqrt{\pi}\mu^{\mu + \frac{1}{2}}h^{\mu}x^{\mu - \frac{1}{2}}}{\Gamma(\mu)H^{\mu - \frac{1}{2}}\bar{X}^{\mu + \frac{1}{2}}} \\ \times \exp\left(-\frac{2\mu hx}{\bar{X}}\right)I_{\mu - \frac{1}{2}}\left(\frac{2\mu Hx}{\bar{X}}\right), \quad (10)$$

where $I_n(\cdot)$ is the *n*-th order modified Bessel function of the first kind [14] and *h* and *H* are functions of η that depend on the distribution's format. In Format 1 of the $\eta - \mu$ distribution, where $\eta > 0$, the expressions of *h* and *H* are given by $h = \frac{2+\eta^{-1}+\eta}{4}$ and $H = \frac{\eta^{-1}-\eta}{2}$, respectively, whereas in Format 2 $h = \frac{1}{1-\eta^2}$ and $H = \frac{\eta}{1-\eta^2}$. The ECC is then given by [16]

$$C_{s}(\bar{X}) = \frac{W2\sqrt{\pi}}{\ln(2)\Gamma(\mu)h^{\mu}} \sum_{i=0}^{\infty} \frac{2^{-2(\mu+i)}}{i!\Gamma\left(\mu + \frac{1}{2} + i\right)} \times \left(\frac{H}{h}\right)^{2i} G_{2,3}^{3,1}\left(\frac{2\mu h}{\bar{X}}\Big|_{2(\mu+i),0,0}^{0,1}\right).$$
(11)

For the mobile case, using (11) in (6) and solving the resulting integral using [14, eq. (7.811.2)], the ECC is obtained as

$$C_m(\bar{\gamma}_D) = \frac{W2\sqrt{\pi}}{\ln(2)\Gamma(\mu)h^{\mu}a} \sum_{k=1}^n B_k \sum_{i=0}^\infty \frac{2^{-2(\mu+i)}}{i!\Gamma(\mu+\frac{1}{2}+i)} \times \left(\frac{H}{h}\right)^{2i} G_{3,4}^{3,2} \left(\frac{2\mu h}{\bar{\gamma}_D}\Big|_{2(\mu+i),0,0,-v_k}^{1-v_k,0,1}\right).$$
(12)

C. $\alpha - \mu$ FADING CHANNEL

This is a two-parameter multipath fading model that generalizes several well-known distributions such as the Nakagami and Hoyt distributions. The PDF of the instantaneous SNR in the static case is given by [17]

$$f_X(x) = \frac{\alpha \mu^{\mu} x^{\alpha \mu/2-1}}{2\Gamma(\mu)\bar{X}^{\alpha \mu/2}} \exp\left(-\mu\left(\frac{x}{\bar{X}}\right)^{\alpha/2}\right), \quad (13)$$

where α , μ are the distribution's shaping parameters. Assuming $\frac{2}{\alpha} = \frac{r}{s}$, the ECC is given by [17, eq. (8)]

$$C_{s}(\bar{X}) = \frac{W\alpha}{\ln(2)\sqrt{s}\Gamma(\mu)(2\pi)^{\frac{2r+s-3}{2}}} \left(\frac{\mu}{\bar{X}^{\frac{\alpha}{2}}}\right)^{\mu} \times G_{2r,s+2r}^{s+2r,r} \left(\frac{(\mu/s)^{s}}{\bar{X}^{r}}\Big|_{\Delta(s,0),\Delta(r,-\frac{\alpha\mu}{2}),\Delta(r,-\frac{\alpha\mu}{2})}^{\Delta(r,-\frac{\alpha\mu}{2})}\right),$$
(14)

where $\Delta(\ell, \alpha) = \frac{\alpha}{\ell}, \frac{\alpha+1}{\ell}, \dots, \frac{\alpha+\ell-1}{\ell}$. For the mobile case, substituting (14) in (6) and making the change of variables $x = t^r$, the ECC is given by

$$C_{m}(\bar{\gamma}_{D}) = \frac{W\alpha\mu^{\mu}/\ln(2)}{\sqrt{s}\Gamma(\mu)(2\pi)^{\frac{2r+s-3}{2}}a\bar{\gamma}_{D}^{\frac{\mu\alpha}{2}}} \sum_{k=1}^{n} \frac{B_{k}}{r} \int_{0}^{1} x^{\frac{\mu\alpha}{2r} + \frac{v_{k}}{r} - 1} \\ \times G_{2r,s+2r}^{s+2r,r} \left(\frac{(\mu/s)^{s}x}{\bar{\gamma}_{D}^{r}}\Big|_{\Delta(s,0),\Delta(r,-\frac{\alpha\mu}{2}),\Delta(r,-\frac{\alpha\mu}{2})}^{\Delta(r,-\frac{\alpha\mu}{2})}\right) dx.$$
(15)

The above integral can be evaluated with the help of [18, eq. (2.24.2.2)], to obtain

$$C_{m}(\bar{\gamma}_{D}) = \frac{Wr^{-1}\alpha\mu^{\mu}}{\ln(2)\sqrt{s}\Gamma(\mu)(2\pi)^{\frac{2r+s-3}{2}}a\bar{\gamma}_{D}^{\frac{\mu\alpha}{2}}} \sum_{k=1}^{n} B_{k}$$
$$\times G_{2r+1,s+2r+1}^{s+2r,r+1}$$
$$\left(\frac{(\mu/s)^{s}}{s\bar{\gamma}_{D}^{r}}\Big|_{\Delta(s,0),\Delta(r,-\frac{\alpha\mu}{2}),\Delta(r,-\frac{\alpha\mu}{2}),-\frac{\mu\alpha}{2r}-\frac{v_{k}}{r}}\right).$$
(16)

We note that for the special case when $\alpha = 2$ and $\mu = m$ (i.e., Nakagami-*m* fading), using functional relation [14, eq. (9.31.5)], (16) reduces to

$$C_m(\bar{\gamma}_D) = \frac{W}{\ln(2)\Gamma(m)a} \sum_{k=1}^n B_k G_{3,4}^{3,2} \left(\frac{m}{\bar{\gamma}_D}\Big|_{m,0,0,-v_k}^{1-v_k,0,1}\right), \quad (17)$$

which agrees with [10, 30].

D. $\alpha - \eta - \mu$ FADING CHANNEL

The $\alpha - \eta - \mu$ fading model includes the $\eta - \mu$ and $\alpha - \mu$ distributions as special cases. The PDF of the instantaneous SNR is given by [3]

$$f_X(x) = \frac{\sqrt{\pi} \alpha h^{\mu} \mu^{\mu + \frac{1}{2}} x^{\frac{\alpha}{2} \left(\mu + \frac{1}{2}\right) - 1}}{\Gamma(\mu) H^{\mu - \frac{1}{2}} \left(\bar{X}\right)^{\frac{\alpha}{2} \left(\mu + \frac{1}{2}\right)}} \exp\left(-2h\mu\left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right) \times I_{\mu - \frac{1}{2}} \left(2\mu H\left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right).$$
(18)

Assuming $\frac{2}{\alpha} = \frac{r}{s}$, the ECC for the static case is given by [19]

$$C_{s}(\bar{X}) = \frac{W\sqrt{\pi}\sqrt{sr^{-1}}\alpha h^{\mu}\mu^{2\mu}}{\ln(2)(2\pi)^{\frac{s+2r-3}{2}}\bar{X}^{\frac{\alpha\mu}{2}}} \sum_{i=0}^{\infty} \frac{(\mu H/\bar{X})^{2i}}{i!\Gamma(\mu + \frac{1}{2} + i)} \times G_{2r,s+2r}^{s+2r,r} \left(\left(\frac{2h\mu}{s\bar{X}^{\frac{\alpha}{4}}}\right)^{s} \right)^{s}$$

$$\begin{vmatrix} \Delta(r, -\alpha(\mu+i)), \Delta(r, 1-\alpha(\mu+i)) \\ \Delta(s, 0), \Delta(r, -\alpha(\mu+i)), \Delta(r, -\alpha(\mu+i)) \end{vmatrix}.$$
 (19)

For the mobile case, substituting (19) in (6), making the change of variables $x = t^{\frac{\alpha s}{4}}$ and using [18, eq. (2.24.2.2)] to solve the resulting integral, the ECC is given by

$$C_{m}(\bar{\gamma}_{D}) = \frac{W\sqrt{\pi}4\alpha h^{\mu}\mu^{2\mu}}{\ln(2)(2\pi)^{\frac{s+2r-3}{2}}\sqrt{sra\bar{\gamma}_{D}^{\frac{\alpha\mu}{2}}}} \sum_{k=1}^{n} B_{k}$$

$$\times \sum_{i=0}^{\infty} \frac{(\mu H/\bar{\gamma}_{D})^{2i}}{i!\Gamma(\mu+\frac{1}{2}+i)} G_{2r+1,s+2r+1} \left(\left(\frac{2h\mu}{s\bar{\gamma}_{D}^{\frac{\alpha}{4}}} \right)^{s} \right)^{s}$$

$$\Big|_{\Delta(s,0),\Delta(r,-\alpha(\mu+i)),\Delta(r,-\alpha(\mu+i)),-\frac{4}{\alpha s}(v_{k}+\frac{\alpha\mu}{2}+2i)} \right).$$
(20)

E. THE $\alpha - \kappa - \mu$ FADING

The $\alpha - \kappa - \mu$ fading model is a generalized fading model that includes the $\kappa - \mu$ and $\alpha - \mu$ distributions as special cases. The PDF of the instantaneous SNR is given by [3]

$$f_X(x) = \frac{\alpha \mu (1+\kappa)^{\frac{\mu+1}{2}} x^{\frac{\alpha(\mu+1)}{4}-1}}{2\kappa^{\frac{\mu-1}{2}} \exp(\mu\kappa) (\bar{X})^{\frac{\alpha(\mu+1)}{4}}} \\ \times \exp\left(-\mu (1+\kappa) \left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right) \\ \times I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)} \left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{4}}\right).$$
(21)

Assuming $\frac{2}{\alpha} = \frac{r}{s}$, the ECC for the static case is given by [19]

$$C_{s}(\bar{X}) = \frac{W\sqrt{s}r^{-1}\alpha\mu^{\mu}(1+\kappa)^{\mu}}{\ln(2)2(2\pi)^{\frac{s+2r-3}{2}}\exp(\mu\kappa)\bar{X}^{\frac{\alpha\mu}{2}}}$$
$$\times \sum_{i=0}^{\infty} \frac{1}{i!\Gamma(\mu+i)} \left(\mu\sqrt{\frac{\kappa(1+\kappa)}{\bar{X}}}\right)^{2i}$$
$$\times G_{2r,s+2r}^{s+2r,r} \left(\left(\frac{\mu(1+\kappa)}{s\bar{X}^{\frac{\alpha}{2}}}\right)^{s}\right)$$
$$\Big|_{\Delta(s,0),\Delta\left(r,-\frac{\alpha(\mu+i)}{2}\right),\Delta\left(r,-\frac{\alpha(\mu+i)}{2}\right)}\right).$$
(22)

For the mobile case, substituting (22) in (6) and using [18, eq. (2.24.2.2)] to solve the resulting integral, the ECC is given by

$$C_{m}(\bar{\gamma}_{D}) = \frac{W(\sqrt{sr})^{-1}\mu^{\mu}(1+\kappa)^{\mu}}{\ln(2)(2\pi)^{\frac{s+2r-3}{2}}\exp(\mu\kappa)a(\bar{\gamma}_{D})^{\frac{\alpha\mu}{2}}} \\ \times \sum_{k=1}^{n} B_{k} \sum_{i=0}^{\infty} \frac{1}{i!\Gamma(\mu+i)} \left(\mu\sqrt{\frac{\kappa(1+\kappa)}{\bar{\gamma}_{D}}}\right)^{2i} \\ \times G_{2r+1,s+2r+1}^{s+2r+1} \left(\left(\frac{\mu(1+\kappa)}{s(\bar{\gamma}_{D})^{\frac{\alpha}{2}}}\right)^{s} \\ \left|_{\Delta(s,0),\Delta\left(r,-\frac{\alpha(\mu+i)}{2}\right),\Delta\left(r,-\frac{\alpha(\mu+i)}{2}\right),-\frac{2}{\alpha s}(v_{k}+\frac{\alpha\mu}{2}+i)}\right),$$
(23)

which reduces to (16) when $\kappa = 0$ (i.e., $\alpha - \mu$ fading). Moreover, when $\alpha = 2$ (i.e., $\kappa - \mu$ fading), using functional [14, eq. (9.31.5)], (23) reduces to

$$C_{m}(\bar{\gamma}_{D}) = \frac{W}{\ln(2)\Gamma(\mu)\exp(\mu\kappa)a} \sum_{k=1}^{n} B_{k} \sum_{i=0}^{\infty} \frac{(\mu\kappa)^{i}}{(\mu)_{i}i!} \times G_{3,4}^{3,2} \left(\frac{\mu(1+\kappa)}{\bar{\gamma}_{D}}\Big|_{\mu+i,0,0,-v_{k}}^{1-v_{k},0,1}\right),$$
(24)

which agrees with [12, eq. (43)] for the κ - μ fading channel.

F. α-κ-μ SHADOWED FADING CHANNEL

The PDF of the instantaneous SNR is given by [7]

$$f_X(x) = \frac{m^m \alpha}{2c^{\mu} \Gamma(\mu) (\mu \kappa + m)^m \bar{X}} \left(\frac{x}{\bar{X}}\right)^{\frac{\alpha \mu}{2} - 1} \\ \times \exp\left(-\frac{1}{c} \left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right) \\ \times {}_1F_1\left(m; \mu; \frac{\mu \kappa}{c (\mu \kappa + m)} \left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right), \quad (25)$$

where $c = \left(\frac{\Gamma(\mu)(\mu\kappa+m)^m}{m^m\Gamma(\mu+\frac{2}{\alpha})_2F_1(m,\mu+\frac{2}{\alpha};\mu;\frac{\mu\kappa}{(\mu\kappa+m)})}\right)^{\frac{\alpha}{2}}$, and $_1F_1(\cdot)$ denotes the confluent hypergeometric function [14, eq. (9.210)]. Restricting to rational parameters $\frac{2}{\alpha} = \frac{r}{s}$, the ECC is derived in Appendix A as

$$C_{s}(\bar{X}) = \frac{W2m^{m}(2\pi)^{1-s+\frac{1-r}{2}}}{\ln(2)\Gamma(\mu)(\mu\kappa+m)^{m}\sqrt{r}} \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa r^{\mu}}{\mu\kappa+m}\right)^{i} \times G_{2\,s,r+2\,s}^{\,r+2\,s,s} \left(\frac{s^{2\,s}}{\bar{X}^{s}(rc)^{r}}\Big|_{\Delta(s,0),\Delta(s,0),\Delta(r,\mu+i)}^{1-\Delta(s,1),\Delta(s,1)}\right).$$
(26)

This is a new result for the ergodic capacity of the α - κ - μ shadowed fading model in the static scenario. For the mobile case, substituting (26) in (6), making the change of variables $x = t^s$ and using [18, eq. (2.24.2.2)] to solve the resulting integral, the ECC is obtained as

$$C_{m}(\bar{\gamma}_{D}) = \frac{W2m^{m}(2\pi)^{1-s+\frac{1-r}{2}}}{\ln(2)\Gamma(\mu)(\mu\kappa+m)^{m}\sqrt{r}sa} \sum_{k=1}^{n} B_{k}$$

$$\times \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa r^{\mu}}{\mu\kappa+m}\right)^{i}$$

$$\times G_{2s+1,r+2s+1}^{r+2s+1} \left(\frac{s^{2s}}{\bar{\gamma}_{D}^{s}(rc)^{r}}\Big|_{\Delta(s,0),\Delta(s,0)\Delta(r,\mu+i),-\frac{v_{k}}{s}}^{1-\frac{v_{k}}{2},1-\Delta(s,1),\Delta(s,1)}\right).$$
(27)

III. ASYMPTOTIC (HIGH SNR) ANALYSIS

In the large SNR regime, the asymptotic average capacity of a static receiver with average SNR \bar{X} can be evaluated using the capacity loss due to the fading as [15, eq. (8)]

$$\bar{C}_s(\bar{X}) \sim \frac{W}{\ln(2)} \left\{ \ln(\bar{X}) + \bar{\omega} \right\},\tag{28}$$

where parameter $\bar{\omega}$ denotes the capacity loss due to fading [5], which is independent of the SNR and may be evaluated as

$$\bar{\omega} = \frac{d}{d\ell} A F^{(\ell)} \Big|_{\ell=0} = \frac{d}{d\ell} \left\{ \frac{E\left(X^{\ell}\right)}{E(X)^{\ell}} - 1 \right\} \Big|_{\ell=0}, \qquad (29)$$

where AF $^{(\ell)}$ denotes the ℓ - th order amount the fading of the instantaneous SNR. The asymptotic (high SNR) average capacity for the mobile receiver may be obtained by substituting (28) in (6) to give

$$\bar{C}_m(\bar{\gamma}_D) \sim \frac{W}{\ln(2)a} \sum_{k=1}^n B_k \int_0^1 t^{v_k - 1} \left\{ \ln\left(\frac{\bar{\gamma}_D}{t}\right) + \bar{\omega} \right\} dt.$$
(30)

The integral in (30) can be evaluated using [18, eq. (4.253.1)] and, after some simplifications, it results to

$$\bar{C}_m(\bar{\gamma}_D) \sim \frac{W}{\ln(2)a} \sum_{k=1}^n \frac{B_k}{v_k} \left\{ \ln\left(\bar{\gamma}_D\right) + \frac{1}{v_k} + \bar{\omega} \right\}, \qquad (31)$$

where we have used the digamma function $\psi(x) = \Gamma'(x)/\Gamma(x)$ [18, eq. (8.360)] and the fact that $\psi(x + 1) - \psi(x) = 1/x$ [18, eq. (8.365.1)]. Rearranging the terms in (31) we obtain

$$\bar{C}_{m}(\bar{\gamma}_{D}) \sim \frac{W}{\ln(2)} \{\ln(\bar{\gamma}_{D}) + \bar{\omega}\} \sum_{k=1}^{n} \frac{B_{k}}{(\beta_{k}+1)} + \frac{Wa}{\ln(2)} \sum_{k=1}^{n} \frac{B_{k}}{(\beta_{k}+1)^{2}}.$$
(32)

Recognizing from [11, Table I] that for all network dimensions $\sum_{k=1}^{n} \frac{B_k}{(\beta_k+1)} = 1$ for the RWP mobility model, (32) becomes

$$\bar{C}_m(\bar{\gamma}_D) \sim \bar{C}_s(\bar{\gamma}_D) + \frac{Wa}{\ln(2)} \sum_{k=1}^n \frac{B_k}{(\beta_k + 1)^2}.$$
 (33)

We observe that the asymptotic ECC in the mobile case is equal to the ECC in the static case, evaluated also at $\bar{\gamma}_D$, plus a term that depends only on mobility parameters and the path loss exponent. This term accounts for the higher capacity values obtained at received SNRs higher than $\bar{\gamma}_D$ as the user moves inside the coverage area.

Next, in order to obtain the asymptotic ECC in the static and mobile scenarios, we determine parameter $\bar{\omega}$ for the different fading distributions considered in this paper.

A. GENERALIZED-K FADING

The ℓ th moment of the SNR for the generalized-*K* fading model is given by

$$E(X^{\ell}) = \frac{\Gamma(m+\ell)\Gamma(k+\ell)}{\Gamma(m)\Gamma(k)} \left(\frac{\bar{X}}{mk}\right)^{\ell}.$$
 (34)

Moreover, the ℓ th order amount of fading can be expressed as

$$AF^{(\ell)} = \frac{\Gamma(m+\ell)\Gamma(k+\ell)}{\Gamma(m)\Gamma(k)} \left(\frac{1}{mk}\right)^{\ell} - 1.$$
(35)

Then based on the product rule of the differentiation, parameter $\bar{\omega}$ is obtained as

$$\bar{\omega} = \frac{\Gamma(m+\ell)\Gamma(k+\ell)}{\Gamma(m)\Gamma(k)} \left(\frac{1}{mk}\right)^{\ell} \\ \times \left\{\psi(m+\ell) + \psi(k+\ell) - \ln(mk)\right\}\Big|_{\ell=0} \\ = \psi(m) + \psi(k) - \ln(mk).$$
(36)

B. $\eta - \mu$ **FADING**

The ℓ th moment of the SNR for the $\eta - \mu$ fading model may be obtained from (10) as

$$E(X^{\ell}) = \frac{\bar{X}^{\ell} \Gamma(2\mu + \ell)}{\Gamma(2\mu)(2\mu)^{\ell}} \left(1 - \left(\frac{H}{h}\right)^{2}\right)^{-\mu - \ell} \times {}_{2}F_{1}\left(\frac{1}{2} - \frac{\ell}{2}, -\frac{\ell}{2}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2}\right), \quad (37)$$

where

$$E(X) = \bar{X} \left(1 - \left(\frac{H}{h}\right)^2 \right)^{-\mu - 1}$$
(38)

and $_2F_1(\cdot)$ denotes the Gauss hypergeometric function. Moreover, the ℓ th order amount of fading can be expressed as

$$AF^{(\ell)} = \frac{A^{-\ell}\Gamma\left(\mu + \frac{\ell}{2}\right)\Gamma\left(\mu + \frac{\ell}{2} + \frac{1}{2}\right)}{\Gamma(\mu)\Gamma\left(\mu + \frac{1}{2}\right)h^{\mu}\left[1 - (H/h)^{2}\right]^{\mu}} \times {}_{2}F_{1}\left(\frac{1}{2} - \frac{\ell}{2}, -\frac{\ell}{2}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2}\right) - 1,$$
(39)

with $A = \frac{\mu}{h^{\mu}} (1 - (\frac{H}{h})^2)^{-\mu}$. Then based on the product rule of the differentiation we have

$$\begin{split} \bar{\omega} &= \frac{d}{d\ell} A F^{(\ell)} \Big|_{\ell=0} \\ &= \frac{A^{-\ell} \Gamma \left(\mu + \frac{\ell}{2}\right) \Gamma \left(\mu + \frac{\ell}{2} + \frac{1}{2}\right)}{\Gamma(\mu) \Gamma \left(\mu + \frac{1}{2}\right) h^{\mu} \left[1 - (H/h)^{2}\right]^{\mu}} \\ &\times \left\{ \left[-\ln(A) + \frac{1}{2} \psi \left(\mu + \frac{\ell}{2}\right) + \frac{1}{2} \psi \left(\mu + \frac{\ell}{2} + \frac{1}{2}\right) \right] \right. \\ &\times {}_{2}F_{1} \left(\frac{1}{2} - \frac{\ell}{2}, -\frac{\ell}{2}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2} \right) \\ &+ \frac{d}{d\ell} {}_{2}F_{1} \left(\frac{1}{2} - \frac{\ell}{2}, -\frac{\ell}{2}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2} \right) \Big|_{\ell=0} \right\}. \end{split}$$
(40)

Finally, using the result for the derivative of the generalized hypergeometric function with respect to ℓ given in Appendix B, we obtain

$$\bar{\omega} = h^{-\mu} \left(1 - \left(\frac{H}{h}\right)^2 \right)^{-\mu} \times \left\{ -\ln(A) + \frac{1}{2}\psi(\mu) + \frac{1}{2}\psi\left(\mu + \frac{1}{2}\right) \right\}$$

$$-\frac{(H/h)^2}{2(2\mu+3)}{}_3F_2\left(\frac{3}{2},1,1;2,\mu+\frac{3}{2};\left(\frac{H}{h}\right)^2\right)\right\},\quad(41)$$

where ${}_{p}F_{q}(\cdot)$ denotes the generalized hypergeometric function [14, eq. (9.14/1)].

C. $\alpha - \mu$ FADING

For the $\alpha - \mu$ fading model, the ℓ th moment of the SNR is given by

$$E(X^{\ell}) = \frac{\bar{X}^{\ell}}{\mu^{2\ell/\alpha} \Gamma(\mu)} \Gamma\left(\mu + \frac{2\ell}{\alpha}\right).$$
(42)

The corresponding ℓ -th order amount of fading is given by

$$AF^{(\ell)} = \left(\frac{\Gamma(\mu)}{\Gamma(\mu + \frac{2}{\alpha})}\right)^{\ell} \frac{\Gamma(\mu + \frac{2\ell}{\alpha})}{\Gamma(\mu)} - 1.$$
(43)

Then based on the product rule of the differentiation, we obtain

$$\bar{\omega} = \left(\frac{\Gamma(\mu)}{\Gamma(\mu + \frac{2}{\alpha})}\right)^{\ell} \frac{\Gamma(\mu + \frac{2\ell}{\alpha})}{\Gamma(\mu)} \times \left\{\frac{2}{\alpha}\psi\left(\mu + \frac{2\ell}{\alpha}\right) + \ln\left(\frac{\Gamma(\mu)}{\Gamma(\mu + \frac{2}{\alpha})}\right)\right\}\Big|_{\ell=0}$$
$$= \frac{2}{\alpha}\psi(\mu) + \ln\left(\frac{\Gamma(\mu)}{\Gamma(\mu + \frac{2}{\alpha})}\right), \tag{44}$$

which agrees with [10, eq.(77)] when $\alpha = 2$, $\mu = m$ (i.e., Nakagami-*m* fading).

D. $\alpha - \eta - \mu$ FADING

The ℓ th moment of the SNR may be obtained from (18) as

$$\mathbb{E}\left(X^{\ell}\right) = \frac{\sqrt{\pi}\alpha h^{\mu}\mu^{\mu+\frac{1}{2}}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}\bar{X}^{\frac{\alpha}{2}(\mu+\frac{1}{2})}} \int_{0}^{\infty} x^{\ell+\frac{\alpha}{2}\left(\mu+\frac{1}{2}\right)-1} \times \exp\left(-2\mu h\left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right) I_{\mu-\frac{1}{2}}\left(2\mu H\left(\frac{x}{\bar{X}}\right)^{\frac{\alpha}{2}}\right) dx.$$
(45)

Making the change of variables $y = (\frac{x}{\bar{x}})^{\alpha/2}$, writing

$$I_{\mu-\frac{1}{2}}(2\mu Hy) = \sum_{j=0}^{\infty} \frac{(\mu Hy)^{2j+\mu-\frac{1}{2}}}{\Gamma\left(j+\mu+\frac{1}{2}\right)j!},$$
(46)

solving the resulting integral and using the gamma duplication formula, a closed-form expression for the moments is obtained as

$$\mathbb{E}\left(X^{\ell}\right) = \frac{\Gamma(\frac{\ell}{\alpha} + \mu)\Gamma(\frac{\ell}{\alpha} + \mu + \frac{1}{2})\bar{X}^{\ell}}{\Gamma(\mu)\Gamma(\mu + 1/2)h^{\mu}(\mu h)^{\frac{2\ell}{\alpha}}} \times \left(1 - \left(\frac{H}{h}\right)^{2}\right)^{-\left(\mu + \frac{2\ell}{a}\right)} \times {}_{2}F_{1}\left(\frac{1}{2} - \frac{\ell}{\alpha}, -\frac{\ell}{\alpha}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2}\right).$$
(47)

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Moreover, the ℓ th order amount of fading can be expressed as

$$AF^{(\ell)} = \frac{A^{-\ell}\Gamma\left(\mu + \frac{\ell}{\alpha}\right)\Gamma\left(\mu + \frac{\ell}{\alpha} + \frac{1}{2}\right)}{\Gamma(\mu)\Gamma\left(\mu + \frac{1}{2}\right)h^{\mu}\left[1 - (H/h)^{2}\right]^{\mu}} \times {}_{2}F_{1}\left(\frac{1}{2} - \frac{\ell}{\alpha}, -\frac{\ell}{\alpha}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2}\right) - 1, \quad (48)$$

where

1

$$A = \frac{\Gamma\left(\mu + \frac{1}{\alpha}\right)\Gamma\left(\mu + \frac{1}{\alpha} + \frac{1}{2}\right)}{\Gamma(\mu)\Gamma\left(\mu + \frac{1}{2}\right)h^{\mu}\left[1 - (H/h)^{2}\right]^{\mu}} \times {}_{2}F_{1}\left(\frac{1}{2} - \frac{1}{\alpha}, -\frac{1}{\alpha}; \mu + \frac{1}{2}; \left(\frac{H}{h}\right)^{2}\right).$$

Then based on the product rule of the differentiation and using the result in Appendix B, we obtain

$$\bar{\omega} = h^{-\mu} \left(1 - \left(\frac{H}{h}\right)^2 \right)^{-\mu} \left\{ -\ln(A) + \frac{1}{\alpha} \psi(\mu) + \frac{1}{\alpha} \psi(\mu) + \frac{1}{2} \right) - \frac{(H/h)^2}{\alpha(2\mu + 3)} \times {}_{3}F_2 \left(\frac{3}{2}, 1, 1; 2, \mu + \frac{3}{2}; \left(\frac{H}{h}\right)^2 \right) \right\}, \quad (49)$$

which reduces to (41) when $\alpha = 2$ (i.e., special case of η - μ fading channel).

E. $\alpha - \kappa - \mu$ FADING

For the $\alpha - \kappa - \mu$ fading model, the ℓ th moment of the SNR is given as

$$\mathbb{E}\left(X^{\ell}\right) = \frac{\mu\alpha(1+\kappa)^{\frac{\mu+1}{2}}}{2\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\bar{X}^{\frac{\alpha}{4}(\mu+1)}} \int_{0}^{\infty} x^{\ell+\frac{\alpha}{4}(\mu+1)-1} \\ \times \exp\left(-\mu(1+\kappa)\left(\frac{x}{\bar{X}}\right)^{\alpha/2}\right) \\ \times I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\left(\frac{x}{\bar{X}}\right)^{\alpha/4}\right) dx.$$
(50)

Making the change of variables $y = (\frac{x}{\overline{x}})^{\alpha/4}$, writing

$$I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)y}\right) = \sum_{j=0}^{\infty} \frac{\left(\mu\sqrt{\kappa(1+\kappa)y}\right)^{2j+\mu-1}}{\Gamma(j+\mu)j!}$$
(51)

and solving the resulting integral, a closed-form expression for the moments is obtained as

$$\mathbb{E}\left(X^{\ell}\right) = \frac{\Gamma\left(\mu + \frac{2\ell}{\alpha}\right)\exp\left(-\mu\kappa\right)}{\Gamma(\mu)\left(\mu(1+\kappa)\right)^{2\ell/\alpha}} (\bar{X})^{\ell}{}_{1}F_{1}\left(\mu + \frac{2\ell}{\alpha}; \mu; \mu\kappa\right)$$
(52)

Moreover, the ℓ th order amount of fading can be expressed as

$$AF^{(\ell)} = \frac{A^{-\ell}\Gamma\left(\mu + \frac{2\ell}{\alpha}\right)}{\Gamma(\mu)} {}_{1}F_{1}\left(-\frac{2\ell}{\alpha}; \mu; -\mu\kappa\right) - 1, \quad (53)$$

where

$$A = \frac{\Gamma\left(\mu + \frac{2}{\alpha}\right)\exp(-\kappa\mu)}{\Gamma(\mu)} {}_{1}F_{1}\left(\mu + \frac{2}{\alpha}; \mu; \mu\kappa\right).$$

Then based on the product rule of the differentiation and using the result in Appendix B, we obtain

$$\bar{\omega} = \frac{A^{-\ell}\Gamma\left(\mu + \frac{2\ell}{\alpha}\right)}{\Gamma(\mu)} \left\{ -\ln(A) + \psi\left(\frac{\ell}{\alpha} + \mu\right) \right. \\ \left. \times_1 F_1\left(-\frac{2\ell}{\alpha};\mu;\kappa\mu\right) + \frac{d}{d\ell} {}_1F_1\left(-\frac{2\ell}{\alpha};\mu;-\mu\kappa\right) \right\} \right|_{\ell=0} \\ = -\ln(A) + \frac{2}{\alpha} \left\{ \psi\left(\mu\right) + \kappa_2 F_2\left(1,1;2,\mu+1;-\mu\kappa\right) \right\}.$$
(54)

For the special case of κ - μ fading, inserting $\alpha = 2$ in (54), we obtain

$$\bar{\omega} = -\ln(\mu(1+\kappa)) + \psi(\mu) + \kappa_2 F_2(1,1;2,\mu+1;-\mu\kappa).$$
(55)

Finally, we note that (54) reduces to the final result in (44) when $\kappa = 0$ (i.e., special case of α - μ fading channel).

F. α-κ-μ SHADOWED FADING

The ℓ th moment of the SNR for the α - κ - μ shadowed fading model may be obtained as

$$\mathbb{E}(X^{\ell}) = \bar{X}^{\ell} \frac{m^{m} \Gamma(\mu + 2\ell/\alpha)}{\Gamma(\mu)(\mu\kappa + m)^{m}} \times {}_{2}F_{1}\left(m, \mu + 2\ell/\alpha; \mu; \frac{\mu\kappa}{\mu\kappa + m}\right).$$
(56)

Moreover, the ℓ th order amount of fading can be expressed as

$$AF^{(\ell)} = A^{-\ell} \frac{m^m \Gamma\left(\mu + \frac{2\ell}{\alpha}\right) c^{2\ell/\alpha}}{\Gamma(\mu)(\mu\kappa + m)^m} \times {}_2F_1\left(m, \mu + \frac{2\ell}{\alpha}; \mu; \frac{\mu\kappa}{\mu\kappa + m}\right) - 1, \quad (57)$$

where $A = \frac{m^m \Gamma(\mu + 2/\alpha)}{\Gamma(\mu)(\mu\kappa + m)^m} {}_2F_1(m, \mu + \frac{2}{\alpha}; \mu; \frac{\mu\kappa}{\mu\kappa + m})$. Therefore, based on the product rule of differentiation, we obtain

$$\bar{\omega} = -\ln(A) + \frac{2}{\alpha} \left\{ \psi(\mu) + \kappa_3 F_2 \left(m + 1, 1, 1; 2, \mu + 1; -\frac{\mu\kappa}{m} \right) \right\}.$$
 (58)

IV. NUMERICAL RESULTS

In this section, we provide analytical and simulation results to investigate the effect of mobility and fading parameters on the attained ECC. We consider a 2D wireless network with cell coverage radius D = 100 m and path loss exponent a = 3. The RWP mobility parameters in (2) are given by [11, Table I] as n = 3, $B_k = \frac{1}{73}$ [324, -420, 96], and $\beta_k = [1, 3, 5]$.

To validate the derived expressions simulation results are obtained by generating random variables according to the PDF of (2) for the distance between the mobile receiver



FIGURE. 1. Analytical and simulated results for the PDF of the distance in the 2D RWP mobility scenario.

and the transmitting antenna, obtain the corresponding instantaneous received SNRs taking into account the path loss and the fading distribution and finally obtain the ECC using Shannon's formula. The analytical results are compared with Monte Carlo simulations, where 100,000 randomly generated distances for each average SNR value $\bar{\gamma}_D$ were considered. Fig. 1 depicts the theoretical and simulated (histogram) PDF of the distance *R* for the 2D RWP mobility scenario. Almost perfect agreement between analytical and Monte Carlo results is observed. Therefore, the channel capacity in the mobile case is determined as the average of the attained capacity at many random points inside the coverage area. For this reason, it can also be regarded as the average spectral efficiency offered by the base station located at the center of the coverage area, which is an important KPI for the wireless network.

In the next figures, we plot the average channel capacity as a function of the transmit SNR, that is, the average SNR without considering the path loss, defined as P_t/N . For comparison purposes, we also plot the average capacity for a static user located at the center of the coverage area, that is, at distance d = D/2. For the considered values of transmit SNRs, the range of receive edge SNRs, $\bar{\gamma}_D$, is between -10 to 20 dB, whereas the corresponding receive SNRs in the static case at d = D/2 are 9 dB more, i.e., \bar{X} is between -1 to 29 dB, assuming a single-slope path loss model with path loss exponent a = 3. It turns out that the ECC at this fixed distance gives a lower approximation of the ECC in the mobile case for all fading distributions considered.

In Fig. 2, we plot the ECC for Nakagami-*m* and generalized-*K* fading models in both the static and mobile cases. We use light fading conditions for the Nakagami-*m* fading with m = 4 and heavy shadowing with k = 2 for the case of generalized-*K* fading. The results depict the impact of fading parameters on the attained ergodic capacities and the difference between the mobile and static cases, with the latter evaluated at distance equal to half the coverage range. The asymptotic average capacity values are also plotted and found to converge to the exact capacity values for high values of the average SNR. Moreover, simulation results match the ²²



FIGURE. 2. Average capacity versus transmit SNR for Nakagami-*m* and generalized-*K* fading channels.



FIGURE. 3. Average capacity versus transmit SNR for $\alpha - \mu$ fading channels.

analytically obtained values validating the correctness of the derived expressions. In Fig. 3, we plot the ECC assuming $\alpha - \mu$ fading with $\alpha = 4$, $\mu = 10$ and $\alpha = 1$ and $\mu = 2$. As expected, the attained average capacities in both the static and mobile cases increase with the increase of the non-linearity parameter α and the number of multipath clusters μ . Simulation values show perfect agreement with the theoretical values.

In Fig. 4, we plot the ECC for the case of no LOS using $\alpha - \eta - \mu$ fading with $\alpha = 2$, $\eta = 2.5$, $\mu = 1.5$ and the case of LOS using $\alpha - \kappa - \mu$ fading with $\alpha = 2$, $\kappa = 5$, $\mu = 10$. Finally, in Fig. 5, we plot the average channel capacity of $\alpha - \kappa - \mu$ shadowed fading assuming heavy fading and shadowing conditions with ($\alpha = 1.5$, $\kappa = 1$, $\mu = 1$, m = 1) and light fading and shadowing conditions with ($\alpha = 2.5$, $\kappa = 5$, $\mu = 5$, m = 5). These two figures depict the effect of different fading parameter values on the attained average capacity. Moreover, the asymptotic channel capacities converge to the exact ones for high values of the average SNR, validating the VOLUME 3, 2022



FIGURE. 4. Average capacity versus transmit SNR for $\alpha - \eta - \mu$ and $\alpha - \kappa - \mu$ fading channels.



FIGURE. 5. Average capacity versus transmit SNR for $\alpha - \kappa - \mu$ shadowed fading channels.

correctness of the derived results. We observe that the convergence is faster for channels with good fading/shadowing conditions, for which the asymptotic values coincide to the exact ones for transmit SNR as low as 65 dB, which corresponds to average received SNR of 14 dB in the static case and edge SNR of 5 dB in the mobile case. Therefore, the asymptotic expressions can be very helpful in practical scenarios to provide an approximation of the ECC of different generalized fading models in both static and mobile scenarios.

V. CONCLUSIONS

In this paper, we analyzed the impact of user mobility on the ECC of wireless receivers that experience fading, shadowing, and path loss. Using the RWP mobility model, we provided analytical expressions for the exact and asymptotic ergodic capacities in the mobile case under several generalized fading conditions. In the high SNR regime, the ECC in the mobile case was expressed in terms of the ECC in the static case, evaluated at the edge SNR, plus a term that depends on the RWP model parameters and the path loss coefficient and is

independent of the fading. Therefore, as indicated by the asymptotic analysis, the slopes of the ECC in the static and mobile cases are equal. Finally, it turned out that the ECC at a fixed distance, that is half the coverage range used in the RWP model, gives a lower approximation of the ECC in the mobile case for all fading distributions considered.

APPENDIX A EVALUATION OF ECC FOR $\alpha - \kappa - \mu$ SHADOWED FADING IN STATIC CASE

The ECC of the $\alpha - \kappa - \mu$ shadowed fading channel is given by inserting the PDF of (25) in (1), making the change of variables $y = (\frac{x}{\bar{X}})^{\alpha/2}$, and expressing ${}_1F_1(\cdot)$ as infinite summation series, to obtain

$$C_{s}(\bar{X}) = \frac{W2m^{m}}{\ln(2)c^{\mu}\Gamma(\mu)(\mu\kappa+m)^{m}} \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa}{c(\mu\kappa+m)}\right)^{i}$$
$$\times \int_{0}^{\infty} \ln\left(1+\bar{X}y^{\frac{2}{\alpha}}\right) y^{\mu+i-1} \exp\left(-\frac{y}{c}\right) dy. \quad (A-1)$$

Expressing the logarithm as contour integral yields

$$C_{s}(\bar{X}) = \frac{W2m^{m}}{\ln(2)\Gamma(\mu)(\mu\kappa+m)^{m}c^{m}} \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa}{(\mu\kappa+m)}\right)^{i}}{\times \frac{1}{2\pi j} \int_{C} \frac{\Gamma(1+u)\Gamma(-u)\Gamma(-u)\bar{X}^{-u}}{\Gamma(1-u)}}{\times \int_{0}^{\infty} y^{\mu-\frac{2u}{\alpha}+i-1} \exp\left(-\frac{y}{c}\right) dy du.$$
(A-2)

Using [14, eq. (3.351.3)], the inner integral gives

$$\int_0^\infty y^{\mu - \frac{2u}{\alpha} + i - 1} \exp\left(-\frac{y}{c}\right) dy = \Gamma\left(\mu - \frac{2u}{\alpha} + i\right) c^{\frac{-2u}{\alpha}}.$$
(A-3)

Restricting to rational parameters, $\frac{2}{\alpha} = \frac{r}{s}$ and letting $\omega = \frac{u}{s}$ we can rewrite (A-2) as

$$C_{s}(\bar{X}) = \frac{W2m^{m}}{\ln(2)\Gamma(\mu)(\mu\kappa+m)^{m}} \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa}{(\mu\kappa+m)}\right)^{i}$$
$$\times \frac{1}{2\pi j} \int_{C'} \frac{\Gamma(1+s\omega)\Gamma(-s\omega)\Gamma(-s\omega)}{\Gamma(1-s\omega)}$$
$$\times \Gamma(\mu+i-r\omega)\left(\bar{X}c^{\frac{r}{s}}\right)^{-\omega} s \, d\omega. \tag{A-4}$$

Using the Gamma multiplication property $\Gamma(nx) = (2\pi)^{(1-n)/2} n^{nx-1/2} \prod_{q=0}^{n-1} \Gamma(\frac{q}{n} + x)$, the ECC becomes

$$C_{s}(\bar{X}) = \frac{W2m^{m}(2\pi)^{1-\ell+\frac{1-r}{2}}}{\ln(2)\Gamma(\mu)(\mu\kappa+m)^{m}} \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa r^{\mu}}{(\mu\kappa+m)}\right)^{i}$$
$$\times \frac{1}{2\pi j} \int_{C'} \frac{\prod_{q=0}^{s-1}\Gamma\left(\frac{1+q}{s}+\omega\right)\Gamma\left(\frac{q}{s}+\omega\right)\Gamma\left(\frac{q}{s}-\omega\right)}{\prod_{q=0}^{s-1}\Gamma\left(\frac{1+q}{s}+\omega\right)}$$

$$\times \prod_{q=0}^{s-1} \Gamma\left(\frac{\mu+i+q}{r}-\omega\right) \left[\left(\bar{X}c^{\frac{r}{s}}\right)^{s} \frac{r^{r}}{s^{2}s}\right]^{-\omega} d\omega$$

$$= \frac{W2m^{m}(2\pi)^{1-s+\frac{1-r}{2}}}{\ln(2)\Gamma(\mu)(\mu\kappa+m)^{m}\sqrt{r}} \sum_{i=0}^{\infty} \frac{(m)_{i}}{(\mu)_{i}i!} \left(\frac{\mu\kappa r^{\mu}}{\mu\kappa+m}\right)^{i}$$

$$\times G_{r+2s,2s}^{s,r+2s} \left(\bar{X}s\frac{(rc)^{r}}{s^{2}s}\Big|^{1-\Delta(s,0),1-\Delta(s,0),1-\Delta(r,\mu+i)}_{\Delta(s,1),1-\Delta(s,1)}\right).$$
(A-5)

Then, applying relation [14, eq. (9.31.2)] on (A-5), finally results to the expression provided in (26).

APPENDIX B

EVALUATION OF $\frac{d}{da_1}[{}_{p}F_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x)]$

In this appendix, we derive the derivative of the generalized hypergeometric function with respect to any of the numerator parameters and evaluate it when the parameter value equals zero. Without loss of generality, we consider the parameter a_1 . Following [20], we may express the generalized hypergeometric as an infinite series and differentiating term-by-term with respect to a_1 , to give

$$\frac{d}{da_{1}} \left[{}_{p}F_{q} \left(a_{1}, \dots, a_{p}; b_{1}, \dots, b_{q}; x \right) \right] \Big|_{a_{1}=0} \\
= \sum_{i=0}^{\infty} \frac{(a_{2})_{i} \cdots (a_{p})_{i}}{(b_{1})_{i} \cdots (b_{q})_{i}} \frac{x^{i}}{i!} \left[\frac{d}{da_{1}} \left(\frac{\Gamma(a_{1}+1)}{\Gamma(a_{1})} \right) \right] \\
= \sum_{i=0}^{\infty} \frac{(a_{1})_{i} \cdots (a_{p})_{i}}{(b_{1})_{i} \cdots (b_{q})_{i}} \frac{x^{i}}{i!} \left[\psi(a_{1}+i) - \psi(a_{1}) \right], \quad (B-1)$$

where $(a)_k = \Gamma(a+k)/\Gamma(a)$ is the Pochhammer symbol and we have used the fact that $\frac{d(a)_k}{da} = (a)_k [\psi(a+k) - \psi(a)]$. Next, we may use the relation $\frac{1}{x+k} = \frac{(x)_k}{x(x+1)_k}$ to write [14, eq. (3.365.3)]

$$\psi(x+i) - \psi(x) = \sum_{k=0}^{i-1} \frac{(x)_k}{x(x+1)_k}.$$
 (B-2)

Substituting (B-2) in (B-1) and noting that the first term in (B-1) equals zero, we have

$$\frac{d}{da_{1}} \left[{}_{p}F_{q} \left(a_{1}, \dots, a_{p}; b_{1}, \dots, b_{q}; x \right) \right]
= \sum_{i=0}^{\infty} \frac{(a_{1})_{i} \cdots (a_{p})_{i}}{(b_{1})_{i} \cdots (b_{q})_{i}} \frac{x^{i}}{i!} \sum_{k=0}^{i-1} \frac{(a_{1})_{k}}{a_{1}(a_{1}+1)_{k}}
= \frac{1}{a_{1}} \sum_{n=0}^{\infty} \frac{(a_{1})_{n+1} \cdots (a_{p})_{n+1}}{(b_{1})_{n+1} \cdots (b_{q})_{n+1}} \frac{x^{n+1}}{(n+1)!} \sum_{k=0}^{n} \frac{(a_{1})_{k}}{(a_{1}+1)_{k}},
(B-3)$$

where we have made the substitution n = i - 1 to get the second line in (B-3). To simplify (B-3) further, we note that $(a)_{n+1} = a(a+1)_n$ and 1/(n+1) = $(1)_n/(2)_n$ to give $\frac{d}{da_1}[{}_{p}F_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x)] = \frac{a_2\cdots a_p}{b_1\cdots b_q}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(1)_n(a_1+1)_n\cdots (a_p+1)_n(a_1)_k}{(2)_n(b_1+1)_n\cdots (b_q+1)_n(a_1+1)_k}\frac{(1)_k1^kx^n}{k!n!}$. Using the following series rearrangement formula [21, p. 100]

$$\sum_{n=0}^{n} \sum_{k=0}^{n} B(k,n) = \sum_{n=0}^{n} \sum_{k=0}^{n} B(k,n+k)$$
(B-4)

(B-3) can be written as a double infinite series as

$$\frac{d}{da_1} \left[{}_{p}F_q \left(a_1, \dots, a_p; b_1, \dots, b_q; x \right) \right] = \frac{a_2 \cdots a_p x}{b_1 \cdots b_q}$$
$$\sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{(1)_{n+k} (a_1+1)_{n+k} \cdots (a_p+1)_{n+k} (a_1)_k}{(2)_{n+k} (b_1+1)_n \cdots (b_q+1)_n (a_1+1)_k} \frac{(1)_k 1^k x^{n+k}}{k! (n+k)!}$$
(B-5)

Setting the parameter $a_1 = 0$ in (B-5), we observe that all the terms in the inner series vanish except for the case when k = 0. Therefore, we have

$$\frac{d}{da_1} \left[{}_{p}F_q\left(a_1, \dots, a_p; b_1, \dots, b_q; x\right) \right] \Big|_{a_1=0}$$

$$= \frac{a_2 \cdots a_p x}{b_1 \cdots b_q} \sum_{n=0}^{\infty} \frac{(1)_n (1)_n (a_2+1)_n \cdots (a_p+1)_n}{(2)_n (b_1+1)_n \cdots (b_q+1)_n} \frac{x^n}{n!}.$$
(B-6)

Recognizing the series in (B-6) as the generalized hypergeometric function, we have

$$\frac{d}{da_{1}} \left[{}_{p}F_{q} \left(a_{1}, \dots, a_{p}; b_{1}, \dots, b_{q}; x \right) \right] \Big|_{a_{1}=0} \\
= \left[\frac{\prod_{i=2}^{p} a_{i}}{\prod_{i=1}^{q} b_{i}} \right] x \\
\times {}_{p+1}F_{q+1} \left(1, 1, a_{2}+1, \dots, a_{p}+1; 2, b_{1}+1, \dots, b_{q}+1; x \right). \tag{B-7}$$

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